PRICING OF EUROPEAN CURRENCY OPTIONS IN PRESENCE OF THE DYNAMIC INFORMATION COSTS

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(DO NOT CIRCULATE)

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Abstract

Information plays a central role of asset pricing. Its effect on the option price depends on the acquisition time of information. So the information costs are varying by the time: this is the dynamic information costs. This work deals with the impact of imperfections, such as the information asymmetry and the market sentiment on the performance of the currency option pricing models. Using the average squared error, we compare the model of Garman and Kolhagen and the new model in presence of dynamic information costs. So, we propose to present and to test the extended Garman and Kolhagen model in presence of dynamic information costs by using some daily data of futures continuous call on the Eur / USD pair from 2 June 2011 to 03 May 2017. Compared to Garman and Kolhagen (1983), this approach produces more reliable and accurate results for the analysis of currency options.

JEL classification:

Keywords: Currency Options, Garman and Kolhagen model, dynamic information costs, derivatives, imperfection.

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1. Introduction

In the past several decades, use of options for hedging or speculative purposes has become widespread. Firms can use options to mitigate price fluctuations in physical commodities and securities, as well as interest rate risk. Another useful application of options is for foreign exchange in which the underlying instrument is a fixed amount of a foreign currency. The use of foreign exchange options (referred to henceforth as “forex” options) is prevalent among firms who have significant operations outside of the country that rely on currency exchange rates. Further, forex options can be used as a part of an investment portfolio as a speculative instrument. Perhaps the most prominent option pricing method is the Nobel Prize winning Black-Scholes model (1973). The underlying instrument in this model is a non-dividend paying stock with returns that follow a geometric Brownian motion. This model is not sufficient for forex options, however, because the distribution only accounts for the domestic risk-free interest rate. In currency exchange, both the domestic and foreign risk-free rates need to be accounted for, thus making the Black-Scholes model inappropriate for pricing. Interest rate parity dictates that the forward premium must equal the interest rate differential, which is reflected in the Garman-Kohlhagen forex option pricing model (1983).

The option market has an incontestable growth since these studies. Several arguments have been advanced to explain its development. They are bound notably to the episodes of excessive volatility and the financial crises that staked out this era of internationalization and caused a need of cover by more and more flexible means than the traditional techniques. So, the work of Garman and Kholhagen (1983) is considered to be the starting point for a good deal of theoretical research concerning the pricing of currency options. This study and that of Orlin Grabbe (1983) were inspired by the work of Black, Scholes, and Merton. Later, they have developed and upgraded by many extensions such as that of Chesney and Loubergé (1987), Melino and Turnbull (1990), Hilliard and Al (1991), Amin and Jarrow (1991), Heston (1993), Sarwar and Krehbiel (2000), Bollen and Rasiel (2003), Xu (2006), Carr and Wu (2007), Xiao et al (2010), Van Haastrecht and Pelsser (2011), Dim et al (2013) and Swishchuk et al (2014). All of these models have been geared to refining the evaluation of currency options.

The Garman–Kolhagen model is based on the following hypotheses:

H1— Exchange rate changes evolve steadily without knowing sudden jumps.

H2— The exchange on the market is continuous.

H3— Interest rates, both in the domestic and foreign markets, are constant.

H4— the foreign exchange market is perfect: no costs of transaction and information…

H5— Option prices are a function of only one stochastic variable, namely S. Thus, the price of the exchange rate will follow a geometric Brownian movement or a process of Gauss Wiener which can be presented as follows:

$$\frac{dS}{S} = \boldsymbol{\mu dt + \sigma dz,}$$

where:

- *dS*: The evolution of the equity price between *t* and *dt*,
- $\boldsymbol{\mu}$: The mathematical expectation of equity return,
- *dZ*: Gauss – wiener standard process, N (0, dt) ,$\boldsymbol{V\ (dz) =\ \emptyset\ dt,}$
- $\boldsymbol{\sigma}$: The standard deviation of instantaneous equity return, which is and known supposed to be constant.

Also, the model of Garman and Kohlhagen does not take into account the market sentiment.

However, the pioneer model of G-K (as well as its extensions) presents several limits, bound closely to the validity of the hypotheses of the model (the hypothesis of constant interest rate, the hypothesis of constant volatility, absence of transaction, information cots, market sentiment...)

So, many empirical works based on the survey of Garman and Kolhagen model of show the existence of a bias in the theoretical prices that are deducted. Indeed, the hypotheses of Garman and Kolhagen model (1983) are not true on the financial markets, which encourage the emergence of new assessment models which are more suitable and realistic.

This situation brings us back to a set of questions to which we try to explore a preliminary answer in this paper:

i. Can the Garman-Kolhagen modified model reduce the mistakes of evaluation caused by the standard G-K model?
ii. What are the impacts of the introduction of these imperfections on the economic agent behaviour on a financial market?
iii. Are the parameters of the option assessment perfectly observed or measured?

This paper is guided by the analogy of the extended B-S model with dynamic imperfection S. Ben Hamad and H.Elleuch (2008). So, the results of this new model proposed approach appears to be more accurate.

The structure of this paper is as follows. Section 2 is rather descriptive. It briefly review works about the real anomalies in currency options pricing and discuss some of the reason why GK model is limited. Section 3 proposes an extend Garman and Kolhagen model taking into account the dynamic information costs. It presents a new derivation of the standard currency option formula which has not been published before. In the fourth section, empirical results are presented and discussed.

2. Literature review

2.1. Literature review on the currency option assessment

The literature shows that the pricing currency options can be divided into two categories.

On the one hand, interest rates, both in the domestic and foreign markets, are constant. But, in reality the spot exchange rate follows a stochastic differential equation. Garman and Kolhagen (1983) developed their model by assuming that the exchange rate follows a geometric Brownian motion. This hypothesis contradicts reality. So to avoid this limitation, many methodologies for evaluating currency options have been proposed using modifications on the GK model, such as Bollen and Rasiel (2003), Carr and Wu (2007), Melino and Turnbull (1990) Hilliard et al. (1991), Sarwar and Krehbiel (2000), Xiao et al (2010), Dim et al (2013) and Swishchuk (2014)...

On the other hand, Grabbe (1983) has developed a model of pricing currency options with stochastic interest rates. As a result, many extensions of the Grabbe model for currency options assessment have been proposed, such as Amin and Jarrow (1991), Heston (1993), Xu (2006) and Van Haastrecht and Pelsser (2011)...

2.1. Literature review of the information costs

The theory of the market microstructure shows the existence of information costs. They have an important role in the selection of an international portfolio. So, it should be taken into account in the pricing of financial assets models.

As well as the approach of Black and Scholes (1973), the market efficiency is among the important assumptions of the Garman and Kolhagen model (1983). Indeed, the economic agents are perfectly rational. On the other hand, it assumes the lack of imperfections on the financial markets. Thus the price of the underlying asset reflects the availability or arrival of new information, Fama (1991).

There are multiple factors responsible for the imperfections existing on the market. we can refer to the presence of transaction and information costs (Jensen 1978; Grossman and Stiglitz 1980; Gu et al 2012; Longjin et al 2016), the presence of information asymmetry (Leland and Pyle 1977; Allen and Gorton 1993; Kashefi Pour 2016), the restrictions on short selling (Harrison and Kreps 1978; Duffie et al. 2002; Scheinkman and Xiong (2003); Feng and Chan

2016; Bohl et al 2016;…). So, taking into account of these imperfections makes arbitrage operations unprofitable to completely eliminate evaluation errors.

Also, behavioral finance has an effect on the decision-making for the investor. So, it causes inefficient market and makes the rationality of investors unrealistic. Multiple research shows that errors of valuation can result when rational investors confront with other irrational, for example, investors qualified as noise traders, as in De Long and al. (1990.a) and Ramiah et al (2015), or investors with over-confidence, as in Daniel and al. (2002) and Abreu and Brunnermeier (2003).

So, by abandoning the hypothesis of investors' rationality, this research attempts to explain how market sentiment, as well as information asymmetry, could have an influence on the model of Garman and Kolhagen (1983).

Several theoretical and empirical researches proved the importance of the imperfections on option assessment. In fact, the literature on this topic distinguishes between two types of anomalies: the classic and the real anomalies.

- The classical anomalies group the models that question the assumptions related to the evolution of the price of the support asset. We can cite, for example options Assessment Models in Presence of Stochastic Volatility which is studied by Hull and White (1987, 1988) and of Heston (1993). Also, we can cite the Assessment Models with Stochastic Process (Mixed or Hybrid). This problem has been studied by Amin and Jarrow (1991), Bo et al. (2010), and Mi-Hsiu Chiang et al (2016). This study provides a theoretical exploration of currency options pricing under the presence of interest-rate regime shifts and exchange-rate asymmetric jumps.
- While the real anomalies group models that are interested of the assumptions that are related to the market.

The classical models of options pricing overlook the effects of information cost. So, it is appropriate to begin by providing an overview of the different types of information costs and motivations for the investor to seek and pay for the information. We can distinguish:

Costs of search and information acquisition

The work of Grossman and Stiglitz (1980) showed that if the information is expensive so it does not have interest to acquire it. This was the problem formalized by Grossman and Stiglitz (1980) while showing that the costly acquisition of such information requires an equilibrium price that is not entirely revealing.

Easley and O'hara (1987) have shown that true knowledge of the quality of information requires more costs that are related to learning. It can be acquired also through experience.

Bellalah and Jacquillat (1995) and and Bellalah (1999) show that the information costs are indispensable to the collection and to the treatment of this information. In this setting, they introduce a new cost of information.

Costs of information transmission

Marin and Rahi [2000] show that the cost of information transmission is perfectly correlated with the number of informed investors. These transmission costs are high when the information is known by a very small number of investors .Rahi (1995) has shown the importance of the role of the transmission of private information in the allocation of financial assets while comparing the alternative structures of assets in a productive economy. Also, Brockman and Yan (2009) and Gul et al. (2010) found that the volume of transactions is positively correlated with the information.

Costs of adverse selection or asymmetric information

Grossman (1976) and Hellwig (1980) assume that information is heterogeneous. So, there are investors who have specific information about the future performance of the risky asset. Additional works consider the existence of single information held by some investors informed (Grossman and Stiglitz [1980]). The results of Kovalenkov and Vives(2014) extend to endogenous information acquisition and the connections with the Grossman–Stiglitz paradox are highlighted. According to Easley et al (1998), several studies assume that the presence of informed investors in the options market causes a deviation of the call-put parity relation in the direction of private information (Cremers and Govindaraj et al (2015 Goncalves et al (2016)).

Hui Ou-Yangand Weili Wu (2016) demonstrate that when the private signal tends to be perfect, the market converges to strong-form efficiency, and thus informed and uninformed traders have almost homogeneous beliefs about the stock payoff, but there is still significant net trade, rather than no trade as erroneously shown by GS.

Market sentiment

The existing literature focuses on how sentiment can affect option prices, but does not offer a theoretical model of options-based sentiment (Han 2008; Mahani and Poteshman 2008; and Bauer and al. 2009). Their empirical results show that the market sentiment measures extracted from the options market and the underlying have a systematic effect on the options pricing.

3. G-K model extension with dynamic information costs

As part of the options assessment model introduced by Black, Scholes and Merton (1973) where the underlying asset (the stock), there is a continuous distribution of a dividend with a rate $\boldsymbol{\delta^1}$. Although, when the underlying is a foreign currency, there is also a distribution of a continuous dividend in the form of a distribution of an interest with a rate that represents the foreign interest with a rate r_f.

The Garman–Kolhagen model ignores the effects of information cost. The extension of this model is based on the majority of the assumptions of the classical model but with the introduction of a new hypothesis which is based on the presence of dynamic information costs.

In this section, we propose to extend the Garman–Kolhagen model by taking into account the dynamical information costs using the risk-neutral argument of Black and Scholes (1973).

The key insight behind this extension is the lack of transparency and liquidity in some markets which is reflected in the search of costly information. The dynamical concept is based on the fact that the information costs depend on the reception time of information. In other words, the investor who obtains the information earlier reduces his risk.

The traditional option pricing models ignore the effects of information cost. Merton (1987), Mineham and Simons (1995), Bellalah and Jacquillat (1995) and Bellalah (2006) included statistical information costs. Ben Hamad and Elleuch (2008a, 2008b) included dynamical imperfection. All these research works explain some bias in the Black–Scholes model.

3.1. Introduction of the dynamic information costs

So, the impact of dynamic information is similar to the application of an additional discount rate.

$$\boldsymbol{r_1 = r +} \text{ Costs associated with dynamic information,}$$

$$r_1 = r + \lambda\, w(l),$$

where λ is the information costs, l is the temporal advantage, and w(l) is the advantage function such that:

$$w(0) = 0\,,$$

$$\lim_{l\to+\infty} w(l) = 1\,,$$

$$\lim_{l\to-\infty} w(l) = -\infty\,.$$

We assume that:

$$w(l) = 1- e^{-\alpha l},$$

where:

- l: the temporal advantage that it varies between 0 and T (the time remaining until maturity) .The maximum advantage of this is T. That's means that the information is obtained at maturity and this parameter is set to 0.
- α: denotes the advantage coefficient which varies between 0 and 1.

In our case, two functions of dynamic information costs are defined:

$$r_{1d} = r_d + \lambda\, w(l) \text{ and } r_{1f} = r_f + \lambda_1 w(l_1),$$

where:

- r_{1f}: the effective foreign (riskless) interest rate,
- r_{1d}: the effective domestic (riskless) interest rate,
- r_f: the real foreign (riskless) interest rate,
- r_d: the real domestic (riskless) interest rate,
- λ: the information costs related to domestic information,
- λ_1: the information costs related to foreign information,
- $w(l)=\mathbf{1-e^{-\alpha l}}$: the advantage function related to domestic information,
- $w(l_1)=\mathbf{1-e^{-\alpha l1}}$: the advantage function related to foreign information,
- l: the temporal advantage related to domestic information,
- l_1: the temporal advantage related to foreign information,
- α: the advantage coefficient related to domestic information,
- α_1: the advantage coefficient related to foreign information.

3.2. Option pricing

In fact, according to the Black–Scholes model[5] it is possible to create a short position composed of the sale of $\frac{1}{\frac{\partial C(S,T)}{\partial S}}$ of an option against a long position on the shares.

Indeed, following the same approach of Black and Scholes [1973] and replacing $\boldsymbol{Se^{-\delta\tau}}$ of the model of Black, Scholes and Merton by $\boldsymbol{Se^{-\tau r_f}}$ of the model of Garman, taking into account

[5] The derivation of Black–Scholes is founded on the concept of arbitration, which constitutes a covered position formed by a long position (or purchase) on the actions and a short position (or sale) on the options, and vice versa. The composition of the PF of arbitration depends on the type of the option:

- Case of option of purchase "call": the position detained on the action must be inverse to that of the call [purchase (sale) of the call + sale (purchase) of the action].
- Case of sale "option was able to ": the same position detained on the action and on the option of sale [purchase (sale) of the call+ purchase (sale) of the action].

the dynamic information costs, we can deduce that if the price of the active support changes by a small amount $r_f S$, the option will change by $\frac{\partial C(S,T)}{\partial S} rfS$. This cover can be maintained without interruption so that the return of the short position becomes completely independent of the change of the value of the active support, i.e. the return to the covered position becomes sure.

As a result, the change in the long position on the currency is approximately offset by the change in $\frac{1}{\frac{\partial C(S,T)}{\partial S}}$ options.

The value of PF, composed of the purchase of an action and the sale of $\frac{1}{\frac{\partial C(S,T)}{\partial S}}$ options, is:

$$S - \frac{1}{\frac{\partial C(S,T)}{\partial S}} \; C(S,T)$$
.

During one time interval, the change of this position is given by:

$$rf \;\; S - \frac{1}{\frac{\partial C(S,T)}{\partial S}} \;\; rf \;\; C(S,T),$$

where: $rf \;\; C(S,T) = C(S + rf \;\; S, T + rf \;\; t) - C(S,T)$.

Geometric Brownian motion governs the currency spot price: i.e. The differential representation of spot price movements is: $ds = \mu \;\; S \;\; dt + \sigma \;\; S \;\; dt$,

where:

- μ: is the drift of the spot currency price.
- σ: is the volatility of the spot currency price.
- and dz: is the standard Wiener process. It follows a normal distribution with 0 mean and variance equal to *dt*.

Using the differential stochastic calculation and Itô's lemma, we get:

$$\Delta C = \frac{\partial C}{\partial t}\Delta S + \frac{\partial C}{\partial t}dt + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2}dt$$

Guided by the analogy of the extended B-S model with dynamic imperfection of Ben Hamad and Elleuch (2008), the return to the covered position is sure, it must be equal to (r_{1d}-r_{1f}) where r_{1d}: is the effective risk-free domestic interest rate and r_{1f} is the effective risk-free foreign interest rate.

So, to deduce the partial derivative equation, there is a necessary condition. This is the lack of arbitrage opportunity from the continuous change in portfolio composition (it remains a risk-free portfolio).

We get:

$$\left(-\frac{\partial C}{\partial t}-\frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2\right)d\ t=(r_{1d}-r_{1f})\left(-C+\frac{\partial C}{\partial S}S\right)d\ t$$

$$\frac{\partial C}{\partial t}+\frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2+(r_{1d}-r_{1f})\frac{\partial C}{\partial s}S-r_{1d}C=0$$: EDP [1]

This is the partial differential equation in the presence of dynamic information costs.

The problem is to solve this partial differential equation. So, the value of the purchase option must fill the condition to the boundary-mark, expressing the value of the call at the date of maturity:

$$C(S, t^*) = \text{Max}\ [0, S\ (t^*) - K].$$

The search for the solution requires a change of variables that leads to the equation of heat.

Let's start by writing the price of the option in the following form:

$$C(S,t)=f(t)y(u_1,u_2),$$

where: $f(t)\ et\ y(u_1,u_2)$ are unknown functions.

So, we calculate the derivative of the option price with respect to time and compared to the price of the underlying asset:

$$\frac{\partial C}{\partial t}=\frac{\partial f}{\partial t}y+f\frac{\partial y}{\partial u_1}\frac{\partial u_1}{\partial t}+f\frac{\partial y}{\partial u_2}\frac{\partial u_2}{\partial t}$$

$$\frac{\partial C}{\partial S}=f\left(\frac{\partial y}{\partial u_1}\frac{\partial u_1}{\partial S}+\frac{\partial y}{\partial u_2}\frac{\partial u_2}{\partial S}\right)$$

$$\frac{\partial^2 C}{\partial S^2}=f\left(\frac{\partial^2 y}{\partial u_1^2}\left(\frac{\partial u_1}{\partial S}\right)^2+\frac{\partial y}{\partial u_1}\frac{\partial^2 u_1}{\partial S^2}\right)+f\left(\frac{\partial^2 y}{\partial u_2^2}\left(\frac{\partial u_2}{\partial y}\right)^2+\frac{\partial y}{\partial u_2}\frac{\partial^2 u_2}{\partial S^2}\right)+2f\left(\frac{\partial^2 y}{\partial u_1\partial u_2}\frac{\partial u_2}{\partial S}\frac{\partial u_1}{\partial S}\right).$$

Then, let's replace these derivatives in equation [1]. We get:

$$\frac{1}{2}\sigma^2 S^2 f\left(\frac{\partial^2 y}{\partial u_1^2}\left(\frac{\partial u_1}{\partial S}\right)^2+\frac{\partial y}{\partial u_1}\frac{\partial^2 u_1}{\partial S^2}\right)+\frac{1}{2}\sigma^2 S^2 f\left(\frac{\partial^2 y}{\partial u_2^2}\left(\frac{\partial u_2}{\partial y}\right)^2+\frac{\partial y}{\partial u_2}\frac{\partial^2 u_2}{\partial S^2}\right)+$$
$$\sigma^2 S^2 f\left(\frac{\partial^2 y}{\partial u_1\partial u_2}\frac{\partial u_2}{\partial S}\frac{\partial u_1}{\partial S}\right)+(r_{1d}-r_{1f})Sf\left(\frac{\partial y}{\partial u_1}\frac{\partial u_1}{\partial S}+\frac{\partial y}{\partial u_2}\frac{\partial u_2}{\partial S}\right)+\frac{\partial f}{\partial t}y+f\frac{\partial y}{\partial u_1}\frac{\partial u_1}{\partial t}+$$
$$f\frac{\partial y}{\partial u_2}\frac{\partial u_2}{\partial t}-r_{1d}\,fy=0\quad [2]$$

Now, it's possible to solve equation [2] by using the « Head Transfer » equation for the function y: $\frac{\partial y}{\partial u_2}=\frac{\partial^2 y}{\partial u_1^2}$

Let's search now $f(t),\ u_1(S,t),\ u_2(S,t)$, so:

$$-r_{1d}\,fy+\frac{\partial f}{\partial t}y=0\ \Rightarrow\ r_{1d}\,f=\frac{\partial f}{\partial t}\Rightarrow f(t)=e^{r_{1d}T}.$$

Let's designate by: $a^2 = \frac{1}{2}\sigma^2 S^2 \left(\frac{\partial u_1}{\partial S}\right)^2$, simplify by: $f(t)$ and rewrite the equation to identify the different terms.

So, $\frac{1}{2}\sigma^2 S^2 \left(\frac{\partial^2 y}{\partial u_1^2}\left(\frac{\partial u_1}{\partial S}\right)^2\right) + \frac{1}{2}\sigma^2 S^2 \left(\frac{\partial y}{\partial u_2}\frac{\partial^2 u_2}{\partial S^2}\right) + (r_{1d} - r_{1f})S\left(\frac{\partial y}{\partial u_2}\frac{\partial u_2}{\partial S}\right) + \frac{\partial y}{\partial u_2}\frac{\partial u_2}{\partial t}$ is cancelled when:

$$-a^2 = \frac{1}{2}\sigma^2 S^2 \left(\frac{\partial^2 u_2}{\partial S^2}\right) + (r_{1d} - r_{1f})S\left(\frac{\partial u_2}{\partial S}\right) + \frac{\partial u_2}{\partial t}.$$

Assuming that $\frac{\partial u_2}{\partial S} = \frac{\partial^2 u_2}{\partial S^2} = 0$, we find that: $\frac{\partial u_2}{\partial t} = -a^2 \Rightarrow u_2 = -a^2 T$ [3].

In these conditions, the EDP is written as follows:

$$\frac{1}{2}\sigma^2 S^2 \left(\frac{\partial^2 u_1}{\partial S^2}\right) + (r_{1d} - r_{1f})S\left(\frac{\partial u_1}{\partial S}\right) + \frac{\partial u_1}{\partial t} = 0 \quad [4]$$

.

Using the expression of a^2 , we get:

$$\frac{\partial u_1}{\partial S} = \sqrt{2}\frac{a}{\sigma}\frac{1}{S}, \text{ so: } u_1 = \sqrt{2}\frac{a}{\sigma}\ln\left(\frac{S}{E}\right) + b(t) \quad [5]$$

Replacing [5] in [4], we get: $\frac{1}{2}\sigma^2 S^2 \left(-\frac{1}{S^2}\sqrt{2}\frac{a}{\sigma} + \frac{\partial b}{\partial t}\right) + (r_{1d} - r_{1f})S\sqrt{2}\frac{a}{\sigma}\frac{1}{S} = 0$

But: $-\frac{1}{2}\sigma\, a\sqrt{2} + \frac{\partial b}{\partial t} + (r_{1d} - r_{1f})\frac{a}{\sigma}\sqrt{2} = 0$

$$\Rightarrow \frac{\partial b}{\partial t} = \frac{a\sqrt{2}}{\sigma}\left(\frac{\sigma^2}{2} - (r_{1d} - r_{1f})\right) \Rightarrow b(t) = \frac{a\sqrt{2}}{\sigma}\left(\frac{\sigma^2}{2} - (r_{1d} - r_{1f})\right)T$$

$$\Rightarrow u_1 = \frac{a\sqrt{2}}{\sigma}\left[\ln\frac{S}{E} + \left(\frac{\sigma^2}{2} - (r_{1d} - r_{1f})\right)T\right]$$

The fact that: $C(S,t^*) = f(t)y(u_1,u_2)$ and $\frac{\partial y}{\partial u_2} = \frac{\partial^2 y}{\partial u_1^2}$, the result at maturity is written as follows:

$$C(S,t^*) = \left[y\left(u_1(S,t^*), u_2(S,t^*)\right)\right] = y\left[\frac{a\sqrt{2}}{\sigma}\ln\frac{S}{E},\ 0\right]$$

The solution of the Head Transfer equation is of the following form:

$$y(u_1,u_2) = \frac{1}{\sqrt{2\Pi u_2}}\int_{-\infty}^{+\infty} u_0(\in) e^{\frac{-\varepsilon^2}{2u_2}} d\varepsilon$$

So, the value of the call at the date of maturity is written as follows:

$C(S,t^*) = y(k,0) = E\left[e^{\frac{\sigma k}{a\sqrt{2}}} - 1\right]$, if k ≥ 0 and 0 if not, where: $k = \frac{\sqrt{2}a}{\sigma} \ln\left(\frac{S}{E}\right)$.

Fianally, we can drift the formula of the price of the Currency European call:

$C = Se^{-r_{1f}T} N(d'_1) - Xe^{-r_{1d}T} N(d'_2)$,

with:

$$d'_1 = \frac{\ln\frac{S}{K} + \left((r_{1d} - r_{1f}) + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \text{ and } d'_2 = d'_1 - \sigma\sqrt{T}.$$

3.3. The new parity relationship and the price of a currency European put

The parity relationship in the presence of dynamic information costs is written as follows:

$$C_t - P_t = Se^{-r_{1f}T} + Xe^{-r_{1d}T},$$

where:

- C: Price of call,
- P: Price of put,
- T:Time to maturity,
- r_{1f}: the effective foreign (riskless) interest rate,
- r_{1d}: the effective domestic (riskless) interest rate.

From this relationship, we can deduct the value of European currency put in the presence of dynamic information costs.

$$C_t - P_t = Se^{-r_{1f}T} + Xe^{-r1_dT},$$

$$C_t = S_t e^{-r_{1f}T} N(d'_1) - Xe^{-r_{1d}T} N(d'_2).$$

So, we can write:

$$S_t e^{-r_{1f}T} N(d'_1) - X_t e^{-r_{1d}T} N(d'_2) - P_t = Se^{-r_{1f}T} - X_t e^{-r_{1d}T},$$

$$\Rightarrow P_t = -S_t e^{-r_{1f}T}[1 - N(d'_1)] + Xe^{-r_{1d}T}[1 - N(d'_2)].$$

We know that: $N(d') + N(-d') = 1$.

So, the formula of the price of the currency European put is written as follows:

$$P_t = S_t e^{-r_{1f}T} N(-d'_1) + Xe^{-r_{1d}T} N(-d'_2),$$

with: $d'_1 = \frac{\ln\frac{S}{K} + \left((r_{1d} - r_{1f}) + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$ and $d'_2 = d'_1 - \sigma\sqrt{T}$

4. Empirical methodology

In this section we give empirical tests of the modified Garman and Kolhagen in the Presence of dynamical Asymmetry and sentiment, which takes into account the dynamical imperfection. More precisely, we are interested in the real anomalies using some daily data relating to call of currency option in the Russian market. Of this fact, we will discuss some dynamic real anomalies: we are going to test the model of assessment of the options in presence of dynamic imperfections (the information asymmetry and the market sentiment).

As well as the Black-Scholes model, the Garman-Kolhagen model is based on the hypothesis that the cover is achieved at once. According to this model, the value of the option only depends on price of the active S support and the time t and the other variables are assumed to be constant. One of the crucial hypotheses in the standard model of Garman and Kolhagen is that the trade makes itself without interruption (in a continuous way). Also, they assume that economic agents are perfectly rational and the lack of imperfections on the financial markets. So, we modify this restraining hypothesis to explain the effect of dynamic imperfections (the information asymmetry and the market sentiment).

We will adopt an adductive approach. In fact, it consists of empirically testing the relevance of what the researchers found in the financial markets, in particular, the existence of dynamic information costs on the market. The purpose of this validation is to empirically test the ability of new model to approximate at best the market price compared with the basic model of Garman and Kolhagen.

4.1. Sample

Our sample is composed of 1546 observations of futures continuous call on the Eur / USD pair quoted in the Russian Trading System. More precisely we will be interested in the prices of the foreign options extracted of the DATASTREAM data base. These foreign currency options are of European type. We will use some daily data during a study period from le 2 June 2011 to 03 May 2017.

4.2. Variables

To implement the both evaluation formulas, all their parameters (such as stock price: S; the exercise price: E; the time remaining until maturity: T = (t-t *); the risk free domestic interest rate: r_d; the risk free foreign interest rate: r_f and volatility:σ) must be identified:

- Underlying price (exchange rate Eur/USD), continuous series - datatype (OU): The price of the underlying series at the close of the option market because it reflects the activity of the day.

- Option strike price - datatype (OS): Displays the option strike price nearest the money.
- 3 Month implied volatility constant maturity - datatype (O3): Displays implied volatility At-the-Money (ATM) with a constant time to maturity of 90 days.
- Market price – datatype (OM): Displays the market price of the option series nearest the money.
- The number of days remaining to maturity is a contract characteristic and therefore it is known. We calculate: T = Number of days remaining until maturity divided by 360.
- The domestic interest rate is given by the « Euribor »with a constant time to maturity of 90 days.
- The foreign interest rate is given by the « Libor » with a constant time to maturity of 90 days.

4.3. Objective

The objective of our empirical approach is to determine if the modified model in the presence of dynamic information costs is a better approximation of the market of currency option prices that is obtained using the Garman-Kolhagen model [1983]. Specifically, our work aims to answer the following questions:

- Can the Garman-Kolhagen modified model reduce the mistakes of evaluation caused by the standard G-K model?
- What are the impacts of the introduction of these imperfections on the economic agent behaviour on a financial market?

Our approach will be organized as follows:

- Calculate the price of the call currency option by the two models.
- Study the performance of models through the analysis of the valuation gaps
- Finally, Study the impacts of the introduction of these dynamic imperfections on the economic agent behaviour on a financial market.

So the fundamental objective of our research work is to test empirically in what measure the hold in account of the dynamic imperfections permit to reduce the mistake of measure committed by the traditional model (the one of Garman & Kolhagen: GK) and to improve the management of risk of a portfolio of foreign options.

To answer our problem and to reach our objective, our empirical study is organized in the following way: First, we define the null hypothesis. Afterwards, we carry out the null hypothesis test. And finally we comment on the results obtained.

To test the accuracy of the model of the currency options assessment to describe the prices observed on the market.

4.4. Hypotheses

The objective pursued by our empiric gait is to determine if the model in presence of dynamic information costs, constitute a better approximation of the true model of assessment of a currency option on indication that the one gotten with the help of the model in the Garman-Kolhagen.

One can formalize the tested hypothesis as follows; either:

- GK: The formula giving the value of the European call currency option on indication with the help of the model in the Garman-Kolhagen.
- PI: The formula giving the value of the European call currency option on indication with the help of the model in presence of dynamic information costs.
- VM: Value of market of the option coted on the market of the European call currency option.

4.5. Comparison between classical and modified G-K model:

We define:

- ✓ EGK = GK –VM: Deviation of the price obtained by the Garman and Kolhagen model [1983] compared to the prices of market of the currency options.
- ✓ EPI = PI –VM: deviation of the price obtained by the model in presence of dynamic information costs to the prices of market of the currency options

If the difference is positive (negative), it means that the option is overvalued (undervalued). So, the theoretical model Of G-K (1983) overestimate or overstate (understate or underestimate) the market price. These differences are calculated for each call every sample day in order to calculate the average error evaluation for each call of the sample. By calculating the squared deviations average for the entire sample of currency call, we obtain the average squared error evaluation EGKM and EPIM respectively:

$$EGKM = (EGK^2),$$

$$EPIM = (EPI^2).$$

To pretend that the model in presence of dynamic information costs constitutes a better approximation of the values of market of the currency options on average that those obtained with the help of model of G-K, comes back to test the hopeless hypothesis:

$$Ho: EGKM - EPIM > 0.$$

It seems important to us to fix what we would be in right to wait for this type of test considering the manner with which the question is asked from now on:

- ✓ **Case of dismissal of H0**: several reasons can be invoked:

- The model in presence of dynamic information costs is either equivalent or less efficient than the model in Garman-Kolhagen.

- The model in presence of dynamic information costs and/or models it in the Garman-Kolhagen are estimated badly, this has led to distorting by the measure of the variable EGKM and EPIM.

- The data are of bad quality (synchronization of the value of the foreign option and that of the underlying asset: the exchange rate, simultaneity of the measure of interest rate...).

- ✓ **Case of non-dismissal of H0**: under reserves that the two models are estimated appropriately, one can think that the model in presence of dynamic information costs provides a better approximation of the value of currency option market on average on indication that the model in the G-K.

To go farther on the one hand in the comparison between the model in presence of dynamic information costs and the model of G-K, we use the VREM, which is defined as follows:

$$VREM = \frac{EGKM - EPIM}{EGKM},$$

where:

- EGKM: the average errors of the model of Garman and Kolhagen (1983).
- EPIM: the average errors of the model in the presence of dynamic information costs.
- VREM: the relative deviation of the average errors of the model in presence of dynamic information costs in relation to the theoretical model of G-K (1983).

4.6. The impacts of the introduction of these imperfections on the economic agent behavior on a financial market: Analysis of implied volatility

Implied volatility can be defined as the volatility integrated in the price of an option observed in the market. It's so important because it determines the market consensus about the probable volatility of underlying in the future (Sammers and Poterba [1986]). This volatility reflects market supply and demand in the price of a currency option.

Then, we can study the impact of the introduction of these dynamic information costs in the currency options assessment on the behavior of economic agent while doing a comparative study with the implied volatility extracted from the modified model and that extracted from the basic model.

The implied volatility can be calculated as follows:

- First, the underlying, domestic and foreign interest rates, dynamic information costs, strike price and time to maturity are replaced in the currency options assessment model.
- Next, we equalize the value observed on the market with the value obtained according to the model. Here, only volatility is unknown.
- To calculate this volatility, we use the Newton algorithm which can be simply programmed on Excel via a Visual Basic script "VB".

So, we repeat this procedure every day for the whole sample.

We define:

- VIPGKM: the average of the implied volatility extracted from the model of Garman and Kolhagen (1983).

- VIPIDM: the average of the implied volatility extracted from the model in the presence of dynamic information costs.

To go farther on the one hand in the impacts of the introduction of these imperfections on the economic agent behavior on a financial market, we use the VVIM, which is defined as follows:

$$VVIM = \frac{VIPIDM - VIPGKM}{VIPIDM},$$

where:

- VVIM: Relative deviation of the average of the implied volatility of the model in presence of dynamic information costs in relation to the theoretical model of G-K (1983).

5. Empirical results

5.1. Model performance

For the entire sample, we found that EGKM – EPIM >0, so the hypothesis Ho is confirmed, as shown in Tables 1 and 2.

Table1: Comparison between classic Garman–Kolhagen model and modified model (With Information Costs)

Currency Options	Call (EUR/USD)
EGKM	0,00599156
EPIM	0,00084666
EGKM-EPIM	0,0051449
VREM in %	-85,869045

Table 2: Minimum and maximum of average errors

Average errors	MIN	MAX
EGKM	5,65505E-10	0,148023809
EPIM	6,46652E-14	0,013227028

With reference to these results, we can conclude that the model in presence of dynamic information costs is a better approximation of real CALL prices on average. So, we find that the average errors of the model in presence of information costs go from 6.47E-14 to 0.01, while for the Garman and Kolhagen model [1983], they vary from 5.66E-10 to 0.19. To arrive at a clearer result, we have calculate the variation of the average error of Garman and Kolhagen model (1983) compared to that of presence of information costs (VREM). We found that the relative variation of the average errors is of the order of -86%.

These results are graphically shown.

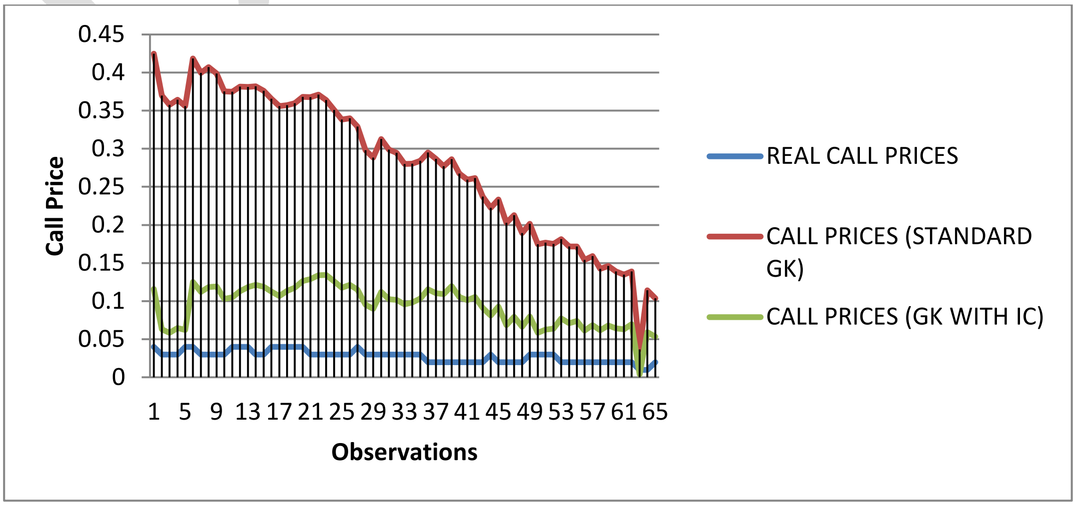

Figure 1: The curves of CALL prices (calculated (GK with information costs), real call prices, calculated (standard GK)) for our Sample from 02/06/2011 to 01/09/2011

The graphs show the curves of Call prices calculated with the two models and the curves of real Call values (market prices). From the graphs, the values of Call market prices are nearer to those calculated with the model with information costs than those calculated with the standard Garman–Kolhagen model.

In other words, taking into account of information cost in the currency call assessment has allowed reducing pricing errors and thus improving the quality of evaluation. This experimental method of curve fitting allows to better estimate the implicit values of λ, λ_1,α,α_1,l and l_1.

We got: λ = 0,001, λ_1= 3,α= 0,01, α_1=0.9, l= 0,2 et l_1=0,393. These implicit values of these parameters ensure a relative variation of the average errors in the order of -86%.

Indeed, in a market where information seems incomplete, information costs play an important role. The investor must determine whether the potential gains are sufficient to ensure the costs of using this strategy. Also, the time to obtain the information is essential to build a profitable strategy. As a result, the introduction of these information costs into the currency options formula generally leads to more accurate theoretical prices since they are included in the currency call option market price.

5.2. Analysis of implied volatility

Table 3: the average error of implied volatility

	Call (EUR/USD)
VIPGKM	0,21791965
VIPIDM	0,48263496
VIPIDM – VIPGKM	0,26471531
VVIM in %	54,847935

As shown in Tables 3, for the entire sample, we found that VIPIDM – VIPGKM >0. We note that the average of implied volatility values is higher for the modified Garman and Kolhagen model than for their classical model (1983). These results are graphically shown.

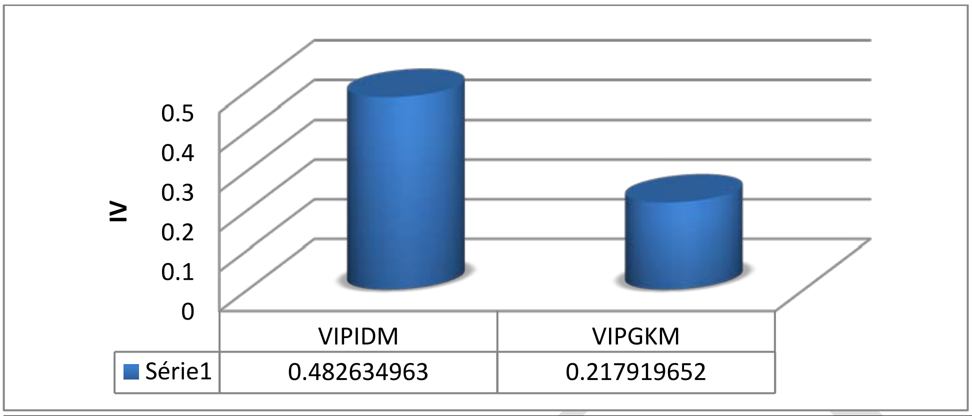

Figure 2: The average of implied volatility

For the entire sample (1547 observations), the values of the implied volatility of the modified model are higher than those of the classical model. It is around 21.79% for the model of Garman and Kolhagen (1983), whereas for the new model with information costs, it is around 48.26% with a variation of 54.85% between the two models.

We can conclude that the introduction of information costs in the valuation of currency European call increased the values of the implied volatility. This means that in the presence of dynamic information costs, the future volatility of the underlying price as perceived by the market is stronger than in the absence of these costs.

These results are consistent with Stein model (1987), which shows that the presence of certain informed agents on the market has a destabilizing effect. As a result, the other agents react because they believe that it is private information. All this leads to an increase to the volatility.

Similarly, Easley and O'hara showed in their study in 1987 that the presence of these uninformed investors usually causes excess volatility. Thus, when considered in the context of asymmetric information, the economic agents predict a higher volatility than that of the expected volatility in a perfect market (symmetric information). Then, the pricing of currency options in presence of information costs assumes informational asymmetry. This may generate higher expected volatility than from the base model wich assumes a perfect market where information asymmetry is non-existent. This explains the observed increase in the values taken by the implied volatility.

6. Conclusion

In this paper, we have developed an extended Garman-Kolhagen model. This model takes into account the dynamical information costs (the information asymmetry and the market sentiment) induced by the dynamical imperfection.

Subsequently, we test empirically the performance of the new model through currency call options on the Eur / USD pair for a period ranging between 2 June 2011 and 03 May 2017. This is done through a comparative analysis of the average squared error of the new model and the G-K model. Also, we studied the impact of the presence of these costs on the behavior of traders through a comparative analysis of the implied volatilities of the new model and the G-K model.

These formulae are simple and have the potential to explain some deviations with respect to the standard Garman–Kolhagen model. The empirical validations show that for the analysis of the currency options, the extended model reproduces more accurate and reliable results.

References

Appendix