The Volatility index and risk aversion: A Detrended Fluctuation Analysis

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Abstract : The financial crisis of the Great Recession of 2007 remembers how difficult it is to anticipate extrem events. In this paper, we want to study the statistical properties of the VIX index to consider the fractal aspect of the risk aversion directly linked to the cycle. Thanks to the Detrended Fluctuation Analysis, we find that the VIX index follows a power law and is characterized by a scale-similarity over the period 1990-2016. This result highlights the existence of a long memory consistent with the non linearity inherent to financial behaviors. The microeconomic foundations of the cycle, indirectly approched by the VIX index evolution, allow to explain its reccurent and regular character. Implicitly, this result has an implication concerning the conduct of the monetary policy.

Keywords : Volatility index, cycle, financial instability, fractals, detrended fluctuation analysis

JEL Codes : G17, G41, N20

1. Introduction

The financial crisis of the Great Recession of 2007 remembers how difficult it is to anticipate extrem events. Several points participate to the underestimation of risks among which the limits of prediction tools and the difficulty to measure financial instability. In this context, a way to improve the analyze of financial stability is to consider behaviors toward risk. Particularly, risk aversion of investors, revealed by asset price movements, can help to anticipate episodes of troubles. Periods of financial tensions would coincide with switchs from risk appetite to risk aversion (Tarashev et al. 2003). The behavior relative to the risk would influence the financial stability whatever the phase of the cycle. In good times, a low risk aversion can induce a sharp increase in both credit and asset prices, which is building the conditions of a future crisis. In bad times, a high risk aversion can entail a higher cost of capital and limit investment.

Several indicators can help to approach risk aversion as simple (VIX, Volatility Index) and aggregate indicators (LCVI, Liquidity, Credit and Volatility Index or GRAI, Global Risk Aversion Index) (Coudert and Gex, 2010). Standard series as gold prices or spreads can also

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reveal tensions on different markets but we choose to consider the VIX index for several reasons[4].

First, the VIX is massively followed by both option traders and equity market participants (Manda, 2008) and it can be considered as benchmark (Whaley, 2009). Consequently, we can say that it reflects the investors behaviors in two ways: from its construction and from its utilization as an indicator of the conjuncture. High VIX levels reveal tensions on financial markets, reflecting a higher need for protection (Sloyer and Tolkin, 2008). Similarly, when the market is relatively stable, VIX levels tend to fall. Given the countercyclical character of the risk aversion, it is natural to concile it with the business cycle (Smith and Whitelaw, 2009 ; Pardo, 2012). Indeed, risk aversion would drives financial and macroeconomic variables (*cf. Animal spirits,* Akerlof and Shiller, 2009). Consequently, we can implicitly include it in the cycles mechanism. In this context, some works draw relationships between the volatility (approached by different ways, not only with the VIX) and key financial and macroeconomic variables (Christiano et al., 2009 ; Bansal et al. 2011 ; Bloom et al., 2014 and Arellano et al., 2016). So, by choosing to analyze the VIX index evolution, we choose to consider an indicator of risk aversion and implicitly a barometer of the cycle. In this paper, instead of analyzing the relationship of the VIX index with some other variables, we prefer to study its statistical properties over the long term.

More particularly, the aim of our study is to analyze the behavior of the VIX time series considering the fractal application. Some other works about the evolution of financial time series converge with our problematic. For example, according to Fernandes et al. (2014), the VIX time series would be far from Gaussian and would present strong evidence of long memory. We expect to recognize some similarities of scales and a long memory in the VIX evolution since it reflects the behavior of investors, wich by essence can't be linear.

This use of long series implies the presence of long memory that can be defined "from an empirical, data-oriented approach in terms of the persistence of observed autocorrelations." (Baillie 1996). Long memory implies the existence of an impulse response of the process to itself with a certain rate of decay. Statistically, stationary, invertible and so short memory process have autocorrelations, which are geometrically bounded. On the contrary, the sum of absolute values of long memory processes autocorrelations has a nonfinite quantity. Moreover, the existence of a long range dependance (Abry et al., 2003) would provide evidence of an endogeneity problem inherent to the evolution of the VIX. Therefore the use of the VIX in linear models to explain other variables is dangerous.

The traditional models of prevision assume that the future follows the past, generally according to the normal distribution. Nevertheless, temporal series are not necessarily identically and independently distributed. The persistence process associated to the long series is accompanied

[4] Note that we prefer to use an implied volatility contrary to simple deviations from means (Konstantinidi et al., 2008). Several studies compared the differences between option implied volatilities and econometric model-based forecasts. The conclusion is that implied volatility has the potential to reflect information that a model-based forecast could not. For example, Becker et al. (2009) found that the VIX index both subsumes information relating to past jump contributions to total volatility and reflects incremental information pertaining to future jump activity. We can also note that the forecast quality of implied volatility of the VIX has improved since its creation (Corrado and Miller, 2005).

by non-stationarity and non-linearity problems. Long series can rather follow a power law (like the prices of cotton, Mandelbrot 2004). Consequently, the intertemporal causality is difficult to implement. Considering this point, non-linear mathematics allow us to take into account the presence of long term correlations in financial series. The Fractal Market theory (Peters, 1994) is based on the application of non-stationary fractional dynamics and the consideration of larger distribution tails. Indeed, it permits to give a greater weight to the occurrence of extreme events (Anderson and Noss 2013). That is why fractal and multifractal models seem to be a good tool to observe the past and anticipate without predicting the future. A fractal approach implies to work with series with a long time horizon to have the ability to capture different scales. Moreover, from a statistical point of view, dealing with long series imply to deal with temporal dependence of varying frequencies (Abry et al., 2003).

2. Data and Methodology

The Chicago Board Options Exchange (CBOE) computes since1993 the VIX index to measure market expectations of the near-term volatility implied by stock index option prices. The VIX index measures the market's expectation of 30-day volatility as implied by the prices of S&P 500 index options without reference to a restrictive option pricing model. The method for calculating the VIX was updated in 2003. The methodology has not changed, but the new VIX uses option prices of the S&P 500 instead of the S&P 100. We choose to work on the VIX series as it gauges the expected market volatility by pooling the information from option prices over the whole volatility skew, not just at-the-money strikes as in the VXO index (Fernandes et al., 2014).

VIX data was downloaded from the CBOE website. The daily data are from January 1990 to February 2016. We have to note that the VIX was introduced as an index in 1993, but we can analyze the historic performance of the VIX since before from Whaley (1993), which retroactively calculated the index to 1986. Our analyze of the volatility index thanks to a high number of observations is helpful to highlight a particular structure of the series (See also Annex 1 for complement). Specifically, we want to determine if the VIX evolution can follow a fractal structure (Mandelbrot, 1982, 1999 ; Calvet et al. 1997a). A fractal structure puts forward notions of self similarity (Cheong, 2010) and scale invariance (Stanley et al., 2000). In this context, we expect to see VIX fluctuations identical from one time scale to another.

To study fluctuations in the VIX time series, we adopt the Detrended Fluctuation Analysis (DFA) (Peng et al., 1994) approach, preferred to the Rescaled Range (R/S) analysis (Hurst, 1951) proposed by Hurst half a century ago. Although DFA is known for producing artifacts for nonlinear trends (Bryce and Sprague, 2012), especially at long term scales, at medium and short term scales it is more or less accepted that the method is more robust when dealing with non-stationary time series[5].

Basically, DFA consists in three steps:

[5] For financial applications, see also Kantelhardt et al., 2002 ; Bolgorian and Gharli, 2010 ; Mali and Mukhopadhyay, 2014.

1) The raw time series *S(k)* is first shifted by its mean *E(s)* and integrated (using a cumulative summation for discrete time series) as follows

$$y(k) = \sum_{i=1}^{k} \big(S(i) - E(S)\big)$$

2) The series *y(k)* is then segmented into windows of various sizes $\delta_n \in \Delta$

3) At each segmentation scale, the integrated data is locally fit to a polynomial function y_{δ_n}(usually a linear function) and the mean-squared residual $F(\delta_n)$ (basically the 'fluctuations') is evaluated as:

$$F(\delta_n) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(y(k) - y_{\delta_n(k)}\right)^2}$$

In the presence of fluctuations in the form of power law, $F(\delta_n) = K(\delta_n)^{\alpha}$, $F(\delta_n)$ increases linearly with increasing δ_n, hence, using a linear regression on the log-log plot of $F(\delta_n)$ one get the slope α which is the scaling exponent of the DFA method, considered as an estimation for the Hurst coefficient.

To check whether function $F(\delta_n)$ follows effectively a power law, one can straightforwardly evaluate the the residual as the root mean squared difference of $log\big(F(\delta_n)\big)$ and $\alpha log(\delta_n) + lok(K)$, namely ,

$$R = \frac{1}{|\Delta|}\sqrt{\sum_{\delta_n \in \Delta}\Big(log\big(F(\delta_n)\big) - \alpha log(\delta_n) - log(K)\Big)^2}$$

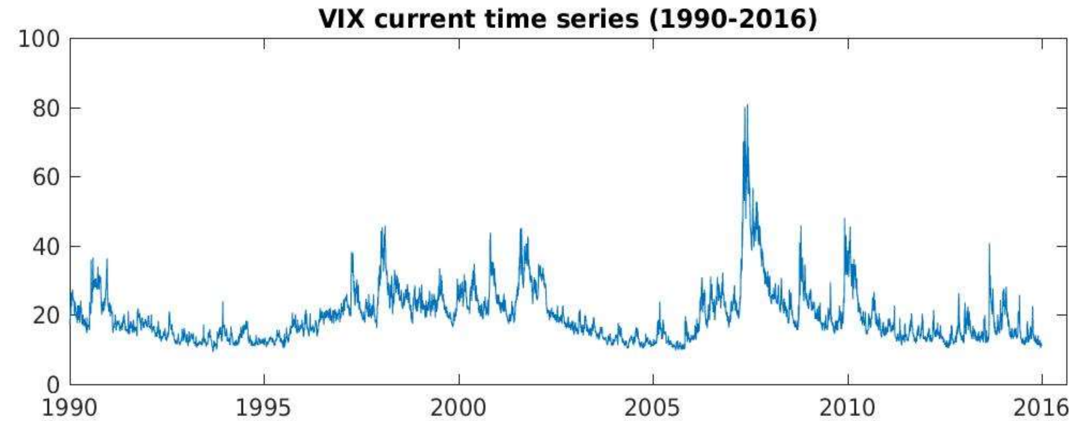
The interpretation of the stochastic process supposedly behind the production of the analyzed time series according to the value of the the scaling coefficient α is the following

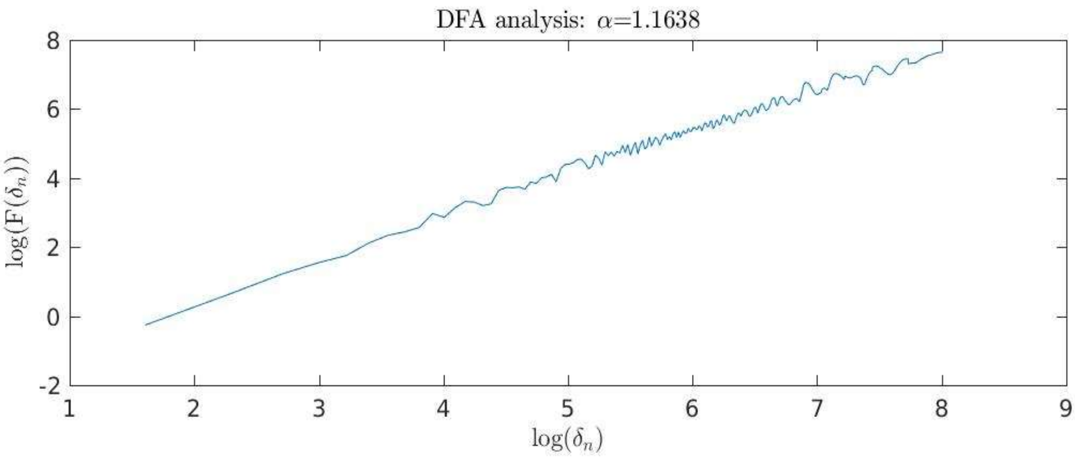
- $\alpha < 1/2$: the process is anti-correlated
- $\alpha \approx 1/2$: the process is uncorrelated, white noise
- $1/2 < \alpha < 1$: the process is correlated
- $\alpha \approx 1$: the process is 1/f-noise, pink noise
- $1 < \alpha < 3/2$: the process is non-stationary, unbounded
- $\alpha \approx 3/2$: the process is Brownian noise

3. Results

The VIX index reaches a minimum value (9,68) in December 2006 (graph 1). The maximum value is reached (80,74) in November 2008 just after the collapse of Lehman Brothers. Its mean value is 19,15. Concerning the scaling coefficient, for VIX current we get: $\alpha = 1.16$ and R=0.005 which shows that the fluctuations follows a clear power law (a marker for self-similar temporal patterns, graph 2) with a very low mean residual. The process behind the VIX process is apparently nonstationnary with unbounded fluctuations.

Graphs 1 & 2 : The VIX time series

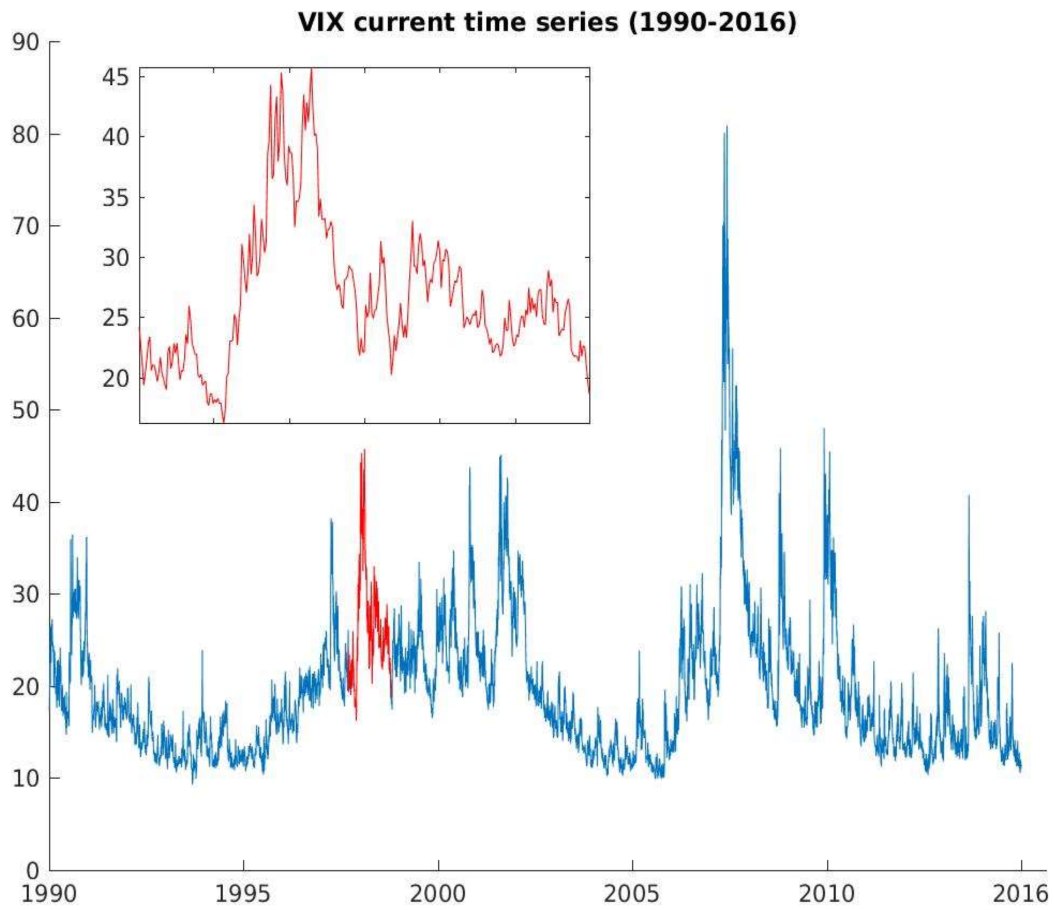




According to the value of α, the existence of a power law implies a long memory effects of the VIX, which is coherent with the power of the financial behaviors behind.

Moreover, we find some similarities of scales in the VIX evolution (graph 3).

Graph 3: A fractal geometry of VIX time series



Same movements occur at different scales of time. These same movements of different magnitude have a cyclical structure described as “waves within waves” or “events within events” (Blackledge 2008). We can note that Mandelbrot (2004) supposes that this cyclical structure of financial series can be explained by expectations. This idea is not new: « Optimism will feed upon *itself and* crystallize so as to become an element of the mechanism of cyclical events and the "cause" of secondary phenomena » (Schumpeter, p.145, 1939). According to Mandelbrot (1999, p.16), « the place of tranquility and mildness is taken by movements that are non-periodic but described by everyone as “cyclic” with many apparent cycle lengths, ranging from very small up to about three cycles in a sample”. Technically, a cycle is said to be non-periodic when it includes trends that persist for irregular periods but with a degree of statistical regularity often associated with non-linear dynamical systems (Blackledge and Rebow 2010). Thus, our fractal analysis of the VIX index permits to highlight non-periodic cycles in long time-series of observations.

4. Conclusion

Considering financial instability leads us to analyze behaviors towards risk. We choose to work with the VIX index time series over the 1990-2016 period in order to capt at least one cycle. Dealing with the time series implies to take into account non linearity and long memory effects. In this context, we adopt a Detrended Fluctuation Analysis highlighting the fractal dimension of the temporal series. According to our results, the VIX index series would follow a power law. Moreover, we can underline its fractal structure over the long term. A great added value of choosing a fractal study applied to a risk aversion and cycle indicator is the readability of the interpretation thanks to the graphical representation. It is easy to observe cycles with statistically self-similar patterns appearing at all the observation scales. Indeed, scale similarity and long memory are inherent to the VIX index evolution. This kind of results coincide with the non linearity of the financial behaviors.

This result has several implications. First, the non linearity of the VIX index series implies to be very cautious whith the predictive models which use the VIX. Second and more generally, the VIX can be chosen as an indicator of the risk aversion and indirectly of the business cycle. Consequently, its microeconomic foundations allow to explain its reccurent, regular but non periodic character. Implicitly, this result has an implication concerning the conduct of the monetary policy. If the financial stability is included in the aim of the monetary policy, all indicators of the cycle are precious. The fact that the VIX index series follows a non-linear process and respect a reccurent movement is an argument to conduct a preemptive monetary policy. Indeed, we can not predict the next return or the next extrem event but we know that it will happen on the long term, highlighting the interest to narrow its amplitude.

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Annex 1 : Temporal series and Self-similarity

Technically, a temporal series is represented on a two-dimensional plane referring to time (on the abscissa) and to the variable of interest (on the y-axis). Therefore, to study the fractal structure of a series, two factors of amplification are required. Statistically, a process $y(t)$ is autosimilar with a parameter α if it has a probability distribution identical to a process re-calibrated by a factor $\frac{t}{a}$:

$$y(t) \stackrel{\text{def}}{=} a^{\alpha} y(\frac{t}{a})$$

With : t the time and $\frac{t}{a}$ the rescaled time

The exponent α, called self-similarity parameter is defined as follows (Abry et al 2003 ; Delignières 2001 ; Morris and Schervish 2002):

We note $M_x = \frac{n_2}{n_1}$ the amplification factor applied to time, that is to say the ratio of the sizes of the initial series n_2 and the extracted series n_1 (voir Figure 1).

We note $M_y = \frac{s_2}{s_1}$ the amplification factor applied to the variable, that is to say the ratio of the standard deviations of the two distributions.

$\alpha = \frac{\ln M_y}{\ln M_x}$ is the slope of the line joining the two points (n_1, s_1) et (n_2, s_2) on a log-log graph.

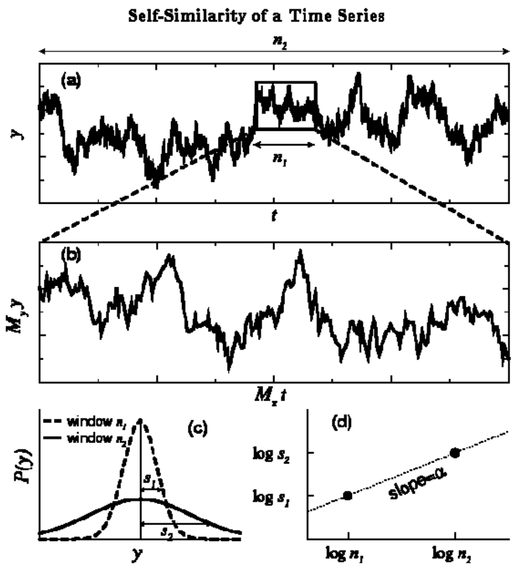

Figure 1 : Self-Similarity of Time Series ; Delignières 2001.