Technology transfer in a linear city with non symmetric locations

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Abstract

This paper compares patent licensing regimes in a Hotelling model where we can find two firms located respectively in the first half of the city and in the second half. I suppose that one of the firms has a process innovation reducing the marginal unit cost. This patent holding firm will decide to license or not the non innovative firm and will choose, when licensing, between a fixed fee or a royalty. The key difference between this paper and other papers is that here I suppose that firms can change their locations on the linear city. I show that when there is no licensing, the non innovative firm makes non positive profit when innovation is drastic. I also show that choice between a fixed fee or a royalty depends on the size of innovation and locations of innovative and non innovative firms. Finally, I find that no licensing is the best strategy for the patent holding firm when innovation is drastic.

Key words: Keywords: Hotelling model, Technology transfer, Patent licensing
JEL Codes: C21, L24, O31, O32

1 Introduction

Several authors studied patent licensing and transfer of innovation. Wang (1998) and (2002) compared licensing regimes in a Cournot duopoly and then in a differentiated Cournot oligopoly and find that optimal licensing regime depends on the size of the innovation (drastic or non drastic). Kamien, Oren and Tauman (1992) studied optimal licensing regime for a cost reducing innovation when innovative firm is outside of the market. Cohen and Morrison

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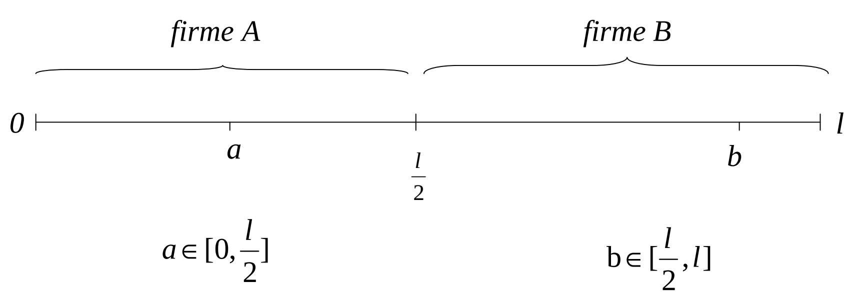
(2004) focused on spillovers in the US food manufacturing industry across states and from agricultural input supply and consumer demand and find average and marginal cost effects in the spatial and industry dimensions that affect location decisions. Mai and Peng (1999) discussed cooperation and competition between firms in a Hotelling spatial model with differentiation. Piga and Theoloky (2005) supposed that R&D spillovers depend on firm's location which means that spillovers increase when firms are close the each others. They show that distance between firm's location increases with the degree of product differentiation. Osborne and Pitchik (1987) studied optimal locations of two competing firms in a Hotelling's model and find that they choose locations close to the quartiles of the market. Paci and Usai (2000) investigate the process of spatial agglomeration of innovation and production activities in an econometric analysis of 85 industrial sectors and 784 Italian local labor systems and find that technological activities of a local industry influence positively innovations of the same sectors in contiguous areas. Alcacer and Chung (2007) examine firms' location choices expecting differences in firm's strategies of new entrants into the United States from 1985 to 1994 and find that firms favor locations with academic innovation activity. Alderighi and Piga (2009) investigate properties of two types of cost reducing restrictions that guarantee the existence of equilibrium in pure strategies in Bayesien spatial models with heterogeneous firms. Poddar and Sinha (2004) studied technology transfer in a Hotelling model where firms are located at the end points of the linear city and find, for an insider patentee, that royalty licensing is optimal when innovation is non drastic while no licensing is the best when innovation is drastic. Matsumura and Matsushima (2008) studied the relationship between licensing activities and the locations of the firms. They find that licensing activities following R&D investment always lead to the maximum differentiation between firms and the mitigation of price competition. Long and Sonbeyran (1998) supposed in a Hotelling model that spillovers depend on the distance between firms and find that agglomeration can be optimal. They also find that geographical dispersion in a two dimensional plane is another possible outcome. Hussler, Lorentz and Rond (2007) supposed that, in a Hotelling model, absorptions capacities of the firms are function of their internal R&D investment and firms determine endogenously the maximum level of knowledge spillovers they might absorb. They find that knowledge spillovers are maximum if firms are located symmetrically and tend to agglomeration in the center of the linear city when transportation cost increase. Pinkse, Slade and Brett (2002) investigate the nature of price competition among firms producing differentiated products and competing in markets that are limited in extent through an econometric study of the US wholesale gasoline markets and find that competition is highly localized. Alderighi and Piga (2008) considered a Salop model with heterogeneous costs and find that cost heterogeneity increases welfare and induce less excessive entry. This paper tries to study optimal licensing regimes when firms are not located at the end points of the linear city (Poddar and Sinha 2004) but are dynamic. The major result is that royalty is not always optimal when innovation is not drastic since non innovative firm makes a non positive profit when the size of innovation is small and so, we cannot find any regime of licensing in this case. Innovative firm will not license its innovation and become a monopoly. Another result shows that no licensing is better that fixed fee licensing independently of the size of innovation.

2 Model

Let's suppose a linear city with a long l and two firms A and B producing homogeneous goods and located respectively on the first half and the second half of the city. The firm A is located at a ($0 < a < \frac{l}{2}$)and the firm B at b ($\frac{l}{2} < b < l$).



To compare patent licensing regimes, I suppose that firm A owns a patented cost reducing innovation allowing to reduce the unit marginal cost by ε which measures the size of the innovation and depends on the investment on R&D by innovative firm. Consumers are uniformly distributed on the linear city (interval $[0, l]$)and each one pay a linear transport cost equal to td (t is the transport unit cost and d the distance between the consumer and the firm). The innovative firm will choose between two licensing regimes : a fixed fee licensing where non innovative firm must pay an amount of money not depending on the quantity produced in exchange of the use of the license or a royalty licensing where non innovative firm must pay a fixed rate on each quantity produced using the new technology. Game stages are as follows: in the first stage, the two firms choose their locations. In the second stage, decides to license its innovation or not and the fixed fee or the royalty to apply and in the third and last stage, the two firms compete in prices. To calculate demand functions of the two firms, we must find the location of the marginal consumer where its utility function when buying the product of the firm A is equal to its utility when buying the product of the firm B. The utility of each consumer depends negatively of the transportation cost and the price of the product.

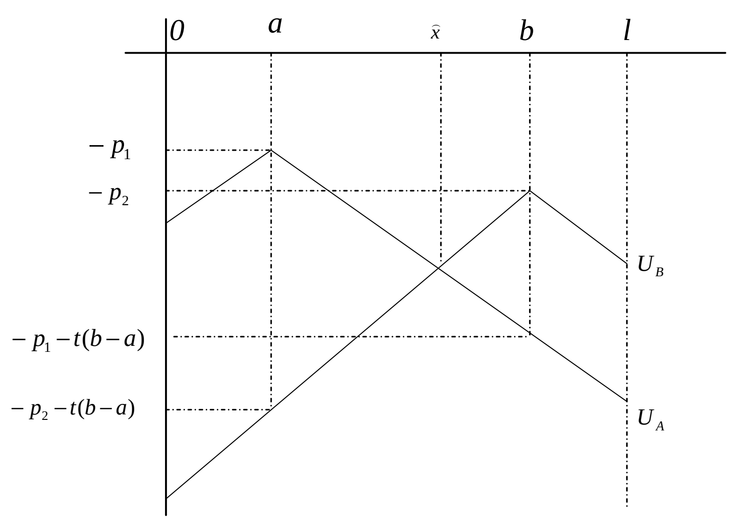
$$U_A = -p_1 - t\,|x-a| \text{ and } U_A = -p_2 - t\,|x-b|$$

The utility function of a consumer located at x and buying the firm A product is :

$$U_A = \begin{cases} -p_1 - t\,(a-x) & if\ \ x < a \\ -p_1 - t\,(x-a) & if\ \ x > a \end{cases}$$

The utility function of a consumer located at x and buying the firm B product is :

$$U_B = \begin{cases} -p_2 - t\,(b-x) & if\ \ x < b \\ -p_2 - t\,(x-b) & if\ \ x > b \end{cases}$$



In the interval $x \in [a, b]$ the location of the marginal consumer is $\tilde{x}$ and verifies $U_A = U_B \Longleftrightarrow \tilde{x} = \frac{p_2 - p_1 + t(a+b)}{2t}$

Demand function of the firm A is :

$$D_A = \begin{cases} l & if\ p_1 \in Int_1^A \\ \tilde{x} & if\ p_1 \in Int_2^A \\ 0 & if\ p_1 \in Int_3^A \end{cases} \Longleftrightarrow D_A = \begin{cases} l & if\ p_1 \in Int_1^A \\ \frac{p_2 - p_1 + t(a+b)}{2t} & if\ p_1 \in Int_2^A \\ 0 & if\ p_1 \in Int_3^A \end{cases}$$

$$\text{where } \begin{array}{l} Int_1^A = [c_1, p_2 - t(b-a)] \\ Int_2^A = [p_2 - t(b-a), p_2 + t(b-a)] \\ Int_3^A = [p_2 + t(b-a), +\infty[\end{array}$$

Demand function of the firm B is :

$$D_B = \begin{cases} 0 & if\ p_2 \in Int_1^B \\ l - \tilde{x} & if\ p_2 \in Int_2^B \\ l & if\ p_2 \in Int_3^B \end{cases} \Longleftrightarrow D_B = \begin{cases} 0 & if\ p_2 \in Int_1^B \\ \frac{p_1 - p_2 + t(2l - a - b)}{2t} & if\ p_2 \in Int_2^B \\ l & if\ p_2 \in Int_3^B \end{cases}$$

$$\text{where } \begin{array}{l} Int_1^B = [p_1 + t(b-a), +\infty[\\ Int_2^B = [p_1 - t(b-a), p_1 + t(b-a)] \\ Int_3^B = [c_2, p_1 - t(b-a)] \end{array}$$

Profits of the innovative and non innovative firms are :

$$\pi_A = \begin{cases} (p_1 - c_1)\, l & if\ p_1 \in Int_1^A \\ (p_1 - c_1)\, \frac{p_2 - p_1 + t(a+b)}{2t} & if\ p_1 \in Int_2^A \\ 0 & if\ p_1 \in Int_3^A \end{cases}$$

$$\pi_B = \begin{cases} 0 & if\ p_2 \in Int_1^B \\ (p_2 - c_2)\, \frac{p_1 - p_2 + t(2l - a - b)}{2t} & if\ p_2 \in Int_2^B \\ (p_2 - c_2)\, l & if\ p_2 \in Int_3^B \end{cases}$$

To find a Nash equilibrium, the prices of the two firms p_1 and p_2 must verify this inequality : $|p_1 - p_2| \leq t(b-a)$. In fact, the profit of the firm A is not positive in the interval INT_3^A and to make a positive profit, firm A should choose a price p_1 verifying $p_1 < p_2 + t(b-a)$.Also, firm B realize a non positive profit in the interval Int_1^B and should choose a price p_2 verifying

$p_2 < p_1 + t(b-a)$. We show finally that a Nash equilibrium exists in the interval Int_2^A (or Int_2^B).

Profits maximization in respect of prices gives:

$$\begin{cases} \frac{\partial \pi_A}{\partial p_1} = 0 \\ \frac{\partial^2 \pi_A}{\partial p_1^2} < 0 \end{cases} \quad \text{and} \quad \begin{cases} \frac{\partial \pi_B}{\partial p_2} = 0 \\ \frac{\partial^2 \pi_B}{\partial p_2^2} < 0 \end{cases}$$

We find at the equilibrium :

$$\begin{cases} \frac{\partial \pi_A}{\partial p_1} = 0 \\ \frac{\partial \pi_B}{\partial p_2} = 0 \end{cases} \iff \begin{cases} p_1 = \frac{1}{2}\left(p_2 + t\left(a+b\right) + c_1\right) \\ p_2 = \frac{1}{2}\left(p_1 + t(2l-a-b) + c_2\right) \end{cases} \Longrightarrow \begin{cases} p_1^* = \frac{1}{3}\left(t(2l+a+b) + 2c_1 + c_2\right) \\ p_2^* = \frac{1}{3}\left(t(4l-a-b) + c_1 + 2c_2\right) \end{cases}$$

The optimal profit functions of firms A and B at the equilibrium are :

$\pi_A^* = \frac{1}{18t}\left(t(2l+a+b) - c_1 + c_2\right)^2$ and $\pi_B^* = \frac{1}{18t}\left(t(4l-a-b) - c_1 + c_2\right)^2$

Demand functions are :

$D_A = \tilde{x} = \frac{1}{6t}\left(2tl + t(a+b) - c_1 + c_2\right)$ *if* $p_1 \in Int_2^A$

$D_B = l - \tilde{x} = \frac{1}{6t}\left(4tl - t(a+b) + c_1 - c_2\right)$ *if* $p_2 \in Int_2^B$

3 No licensing

In this regime, innovative firm profit alone from its innovation while non innovative firm uses the old technology. Denoting by c_1 and c_2 marginal unit costs of respectively firm A and firm B, we can write: $c_1 = c - \varepsilon$ and $c_2 = c$. Replacing in firms equilibrium profits, we find:

$\pi_A^{PL} = \frac{1}{18t}\left(t(2l+a+b) + \varepsilon\right)^2$ and $\pi_B^{PL} = \frac{1}{18t}\left(t(4l-a-b) - \varepsilon\right)^2$

Price equilibrium are :

$p_1^* = \frac{1}{3}\left(t(2l+a+b) + 3c - 2\varepsilon\right)$ and $p_2^* = \frac{1}{3}\left(t(4l-a-b) + 3c - \varepsilon\right)$

Proposition 1 *when firms are located on the first and second half of the linear city and when there is no licensing, the non innovative firm leave the market when innovation is drastic.*

PROOF. $p_2^* = \frac{1}{3}\left(t(4l-a-b) + 3c - \varepsilon\right)$.

$p_2^* > c$ if $\varepsilon < t(4l - a - b$ (non drastic innovation) and $p_2^* < c$ if $\varepsilon > t(4l - a - b)$ (drastic innovation)

Proposition 2 *When innovation is not drastic and not licensed, the two competing firms agglomerate, in the equilibrium, in the center of the city* $a^* = b^* = \frac{l}{2}$

PROOF. $\frac{\partial \pi_A}{\partial a} = \frac{1}{9}\left(t\left(2l + a + b\right) + \varepsilon\right) > 0$*. Since firm A is located in a with* $0 \leq a \leq \frac{l}{2}$ *then optimal location for firm A is* $a^* = \frac{l}{2}$.

$\frac{\partial \pi_B}{\partial b} = -\frac{1}{9}\left(t\left(4l - a - b\right) - \varepsilon\right)$. *Firm B optimal location depends on the size of the innovation. In fact, when innovation is non drastic* $(\varepsilon < t(4l - a - b))$ *then* $\frac{\partial \pi_B}{\partial b} < 0$ *and firm B will locate in* $b^* = \frac{l}{2}$ *since* $\frac{l}{2} \leq b \leq l$*. However, when innovation is drastic* $(\varepsilon > t(4l - a - b))$ *then* $\frac{\partial \pi_B}{\partial b} > 0$ *and* $b^* = l$ *(in this case firm B leave the market since it don't benefit of a license of a drastic innovation)*

Equilibrium profit functions fir a non drastic innovation are :

$\pi_A^{PL} = \frac{1}{18t}\left(3tl + \varepsilon\right)^2$ and $\pi_B^{PL} = \frac{1}{18t}\left(3tl - \varepsilon\right)^2$

4 Fixed fee licensing

In this regime, firm B can use the new technology in exchange of the payment of a fixed fee denoted by F to the patent holding firm. The maximum amount that firm A can choose is equal to the increase of firm B profit when using the new technology. $F = \pi_B^F - \pi_B^{PL} - \alpha$ with $\alpha \longrightarrow 0$ to be sure that firm B will accept to buy the license.

Firm A and firm B production unit costs are $c_1 = c_2 = c - \varepsilon$. Replacing in the profit functions we find : $\pi_A^F = \frac{t}{18}\left(2l + a + b\right)^2$ and $\pi_B^F = \frac{t}{18}\left(4l - a - b\right)^2$

Fixed fee amount is equal to :

$F = \frac{\varepsilon}{9t}\left(4tl - t\left(a + b\right) - \frac{\varepsilon}{2}\right) - \alpha$

Total revenue of the patent holding firm is:

$\Pi_A^F = \pi_A^F + F = \frac{t}{18}\left(2l + a + b\right)^2 + \frac{\varepsilon}{9t}\left(4tl - t\left(a + b\right) - \frac{\varepsilon}{2}\right) - \alpha$

Firm B total profit after the payment of royalties is :

$\pi_B^{-F} = \pi_B^F - F = \frac{t}{18}\left(t(4l-a-b)-\varepsilon\right)^2 - \alpha$

Lemma 3 *In a fixed fee licensing, non innovative and patent holding firm agglomerate in the center of the city when innovation is strong but non drastic. For a small or intermediate innovation, firms are dispersed.*

PROOF. $\frac{\partial \Pi_A^F}{\partial a} = -\frac{1}{9}\left(\varepsilon - t(2l+a+b)\right).$

$\frac{\partial \Pi_A^F}{\partial a} > 0$ *if* $\varepsilon < t(2l+a+b)$ *and* $\frac{\partial \Pi_A^F}{\partial a} < 0$ *if* $\varepsilon > t(2l+a+b)$.*Since* $a \in [0, \frac{l}{2}]$ *then* $a^* = 0$ *if* $\varepsilon < t(2l+a+b)$ *and* $a^* = \frac{l}{2}$ *if* $\varepsilon > t(2l+a+b)$.

$\frac{\partial \pi_B^{-F}}{\partial b} = \frac{t}{9}\left(\varepsilon - t(4l-a-b)\right) < 0$ *because we are working on the non drastic innovation case which means that* $\varepsilon < t(4l-a-b)$. *Since* $b \in [\frac{l}{2}, l]$ *then* $b^* = \frac{l}{2}$

Firm A total revenue can be written as follows:

$$\Pi_A^F = \begin{cases} \frac{1}{2}tl^2 + \frac{1}{3}\varepsilon l - \frac{\varepsilon^2}{18t} - \alpha & \text{if } 0 < \varepsilon < t(2l+a+b) \\ \frac{25}{72}tl^2 + \frac{7}{18}\varepsilon l - \frac{\varepsilon^2}{18t} - \alpha & \text{if } t(2l+a+b) < \varepsilon < t(4l-a-b) \end{cases}$$

	$0 < \varepsilon < t(2l+a+b)$	
PL	$\left(\pi_A^{PL} = \frac{1}{18t}(3tl+\varepsilon)^2, \pi_B^{PL} = \frac{1}{18t}(3tl-\varepsilon)^2\right)$	$a^* = \frac{l}{2}$, $b^* = \frac{l}{2}$
F	$\left(\Pi_A^F = \frac{1}{2}tl^2 + \frac{1}{3}\varepsilon l - \frac{\varepsilon^2}{18t} - \alpha, \pi_B^{-F} = \frac{1}{18t}(3tl-\varepsilon)^2 + \alpha\right)$	$a^* = \frac{l}{2}$, $b^* = \frac{l}{2}$

	$t(2l+a+b) < \varepsilon < t(4l-a-b)$	
PL	$\left(\pi_A^{PL} = \frac{1}{18t}(3tl+\varepsilon)^2, \pi_B^{PL} = \frac{1}{18t}(3tl-\varepsilon)^2\right)$	$a^* = \frac{l}{2}$, $b^* = \frac{l}{2}$
F	$\left(\Pi_A^F = \frac{25}{72}tl^2 + \frac{7}{18}\varepsilon l - \frac{\varepsilon^2}{18t} - \alpha, \pi_B^{-F} = \frac{1}{18t}\left(\frac{7}{2}tl-\varepsilon\right)^2 + \alpha\right)$	$a^* = 0$, $b^* = \frac{l}{2}$

Proposition 4 *No licensing is always better than fixed fee licensing for a patent holding firm on a Hotelling model where firms share equally their locations on the linear city.*

PROOF. $\Pi_A^F - \pi_A^{PL} = -\frac{\varepsilon^2}{9t} - \alpha < 0$ *if* $0 < \varepsilon < t(2l+a+b)$

$\Pi_A^F - \pi_A^{PL} = -\frac{11}{72}tl^2 + \frac{1}{18}\varepsilon\left(l - 2\frac{\varepsilon}{t}\right) - \alpha < 0$ *if* $t(2l+a+b) < \varepsilon < t(4l-a-b)$ *(since here* $l - 2\frac{\varepsilon}{t} < 0$*)*

5 Royalty licensing

In the royalty regime, the cost-reducing innovation is sold to the non innovative firm in exchange of a royalty amount depending on the production made with the use of the new technology. The amount of royalties is proportional to the demand of firm B and equal to $r\left(l-\tilde{x}\right)$. Firm B will accept to buy the license in this regime only when it will allow it to increase its no licensing profit which means that r must be in the interval $]0,\varepsilon[$ unless this licensing regime will not be important to study.

Production unit costs of firms A and B are respectively $c_1 = c-\varepsilon$ and $c_2 = c-\varepsilon+r$. Replacing in the price functions we find :

$p_1^* = \frac{1}{3}\left(t(2l+a+b)+3c-3\varepsilon+r\right)$ and $p_2^* = \frac{1}{3}\left(t(4l-a-b)+3c-3\varepsilon+2r\right)$

profit equilibrium functions are

$\pi_A^r = \frac{1}{18t}\left(t(2l+a+b)+r\right)^2$ and $\pi_B^r = \frac{1}{18t}\left(t(4l-a-b)-r\right)^2$

To find optimal royalty rate for innovative firm, we should find the total revenue of firm A and then maximize it with respect to r. Firm A total revenue is:

$$\Pi_A^r = \pi_A^r + r(l-\tilde{x}) = \frac{1}{18t}\left(t(2l+a+b)+r\right)^2 + \frac{r}{6t}\left(4tl-t\left(a+b\right)-r\right)$$

$$\begin{cases} \frac{\partial \Pi_A^r}{\partial r} = 0 \\ \frac{\partial^2 \Pi_A^r}{\partial r^2} = -\frac{2}{9t} < 0 \end{cases} \Longrightarrow r = 4tl - \frac{t}{4}\left(a+b\right)$$

Since $r < \varepsilon < t(4l-a-b)$ then $r = 4tl - \frac{t}{4}\left(a+b\right) > \varepsilon$ and so $r^* = \varepsilon - \alpha$ (with $\alpha \to 0$). In fact, optimal per unit royalty must not exceed ε. Since Π_A^r is an increasing function of r on the interval $[0,\varepsilon]$ then the per unit royalty maximizing Π_A^r is ε. Firm B will accept to buy a license only when it makes a profit higher than the non licensing regime which means that it will accept this licensing regime when the per unit royalty is lower than ε. We will take the optima per unit royalty $r^* = \varepsilon - \alpha$ where α close to 0.

Total revenues of firm A and firm B after licensing are :

$$\Pi_A^r = \frac{1}{18t}\left(t(2l+a+b)+\varepsilon\right)^2 + \frac{\varepsilon}{6t}\left(4tl-t\left(a+b\right)-\varepsilon\right)$$

$$\pi_B^{-r} = \frac{1}{18t}\left(t(4l-a-b)-\varepsilon\right)^2 - \frac{\varepsilon}{6t}\left(4tl-t\left(a+b\right)-\varepsilon\right)$$

Lemma 5 *In a royalty licensing, firms agglomerate when innovation is small ($a^* = b^* = \frac{l}{2}$ if $0 < \varepsilon < \frac{8}{5}tl - \frac{2}{5}t(a+b)$) and are dispersed for an intermediate innovation ($a^* = \frac{l}{2}$ and $b^* = l$ if $\frac{8}{5}tl - \frac{2}{5}t(a+b) < \varepsilon < 4tl - t(a+b)$).*

PROOF. $\frac{\partial \Pi_A^r}{\partial a} = -\frac{1}{18}\left(\varepsilon - \left(4tl + 2t\left(a+b\right)\right)\right) < 0$ *because* $4tl - t\left(a+b\right) < 4tl + 2t\left(a+b\right)$. *Since* $0 \leq a \leq \frac{l}{2}$ *then* $a^* = \frac{l}{2}$

$\frac{\partial \pi_B^{-r}}{\partial a} = \frac{5}{18}\left(\varepsilon - \left(\frac{8}{5}tl - \frac{2}{5}t\left(a+b\right)\right)\right)$

$\frac{\partial \pi_B^{-r}}{\partial a} < 0$ *if* $\varepsilon < \frac{8}{5}tl - \frac{2}{5}t(a+b)$ *and* $\frac{\partial \pi_B^{-r}}{\partial a} > 0$ *if* $\varepsilon > \frac{8}{5}tl - \frac{2}{5}t(a+b)$ *where* $\frac{l}{2} \leq b \leq l$

Total revenues of innovative and non innovative firms after choosing their locations can be written as :

	$0 < \varepsilon < \frac{8}{5}tl - \frac{2}{5}t\left(a+b\right)$	
PL	$\left(\pi_A^{PL} = \frac{1}{18t}\left(3tl+\varepsilon\right)^2, \pi_B^{PL} = \frac{1}{18t}\left(3tl-\varepsilon\right)^2\right)$	$a^* = b^* = \frac{l}{2}$
R	$\left(\Pi_A^r = \frac{1}{18t}\left(3tl+\varepsilon\right)^2 + \frac{\varepsilon}{6t}\left(3tl-\varepsilon\right), \pi_B^{-r} = \frac{1}{18t}\left(3tl-\varepsilon\right)^2 - \frac{\varepsilon}{6t}\left(3tl-\varepsilon\right)\right)$	$a^* = b^* = \frac{l}{2}$

	$\frac{8}{5}tl - \frac{2}{5}t(a+b) < \varepsilon < 4tl - t(a+b)$	
PL	$\left(\pi_A^{PL} = \frac{1}{18t}\left(3tl+\varepsilon\right)^2, \pi_B^{PL} = \frac{1}{18t}\left(3tl-\varepsilon\right)^2\right)$	$a^* = b^* = \frac{l}{2}$
R	$\left(\Pi_A^r = \frac{1}{18t}\left(\frac{7}{2}tl+\varepsilon\right)^2 + \frac{\varepsilon}{6t}\left(\frac{5}{2}tl-\varepsilon\right), \pi_B^{-r} = \frac{1}{18t}\left(\frac{5}{2}tl-\varepsilon\right)^2 - \frac{\varepsilon}{6t}\left(\frac{5}{2}tl-\varepsilon\right)\right)$	$a^* = \frac{l}{2}$, $b^* = l$

Proposition 6 *Royalties licensing is better than fixed fee licensing for a patent holding firm when innovation is non drastic.*

PROOF. *If* $0 < \varepsilon < \frac{8}{5}tl - \frac{2}{5}t\left(a+b\right)$ *then* $\Pi_A^r - \pi_A^{PL} = \frac{\varepsilon}{6t}\left(3tl-\varepsilon\right) > 0$. *If* $\frac{8}{5}tl - \frac{2}{5}t(a+b) < \varepsilon < 4tl - t(a+b)$ *then* $\Pi_A^r - \pi_A^{PL} = \frac{1}{18t}\left(\frac{7}{2}tl+\varepsilon\right)^2 + \frac{\varepsilon}{6t}\left(\frac{5}{2}tl-\varepsilon\right) - \frac{1}{18t}\left(3tl+\varepsilon\right)^2 > 0$

Lemma 7 *For a small innovation, a Nash equilibrium do not exist for a royalty licensing and then we have not necessarily two competing firms on the linear city.*

PROOF. *See Appendix*

6 Fixed fee versus royalties

Total revenues of patent holding firm under fixed fee licensing and royalties licensing calculated after their optimal positioning on the linear city and depending on the size of innovation are :

	$0<\varepsilon<t(2l+a+b)$	
F	$\left(\Pi_A^F=\frac{1}{2}tl^2+\frac{1}{3}\varepsilon l-\frac{\varepsilon^2}{18t}-\alpha,\pi_B^{-F}=\frac{1}{18t}\left(3tl-\varepsilon\right)^2+\alpha\right)$	$a^*=\frac{l}{2}$, $b^*=\frac{l}{2}$

	$t(2l+a+b)<\varepsilon<t(4l-a-b)$	
F	$\left(\Pi_A^F=\frac{25}{72}tl^2+\frac{7}{18}\varepsilon l-\frac{\varepsilon^2}{18t}-\alpha,\pi_B^{-F}=\frac{1}{18t}\left(\frac{7}{2}tl-\varepsilon\right)^2+\alpha\right)$	$a^*=0$, $b^*=\frac{l}{2}$

	$0<\varepsilon<\frac{8}{5}tl-\frac{2}{5}t\left(a+b\right)$	
R	$\left(\Pi_A^r=\frac{1}{18t}\left(3tl+\varepsilon\right)^2+\frac{\varepsilon}{6t}\left(3tl-\varepsilon\right),\pi_B^{-r}=\frac{1}{18t}\left(3tl-\varepsilon\right)^2-\frac{\varepsilon}{6t}\left(3tl-\varepsilon\right)\right)$	$a^*=b^*=\frac{l}{2}$

	$\frac{8}{5}tl-\frac{2}{5}t(a+b)<\varepsilon<4tl-t(a+b)$	
R	$\left(\Pi_A^r=\frac{1}{18t}\left(\frac{7}{2}tl+\varepsilon\right)^2+\frac{\varepsilon}{6t}\left(\frac{5}{2}tl-\varepsilon\right),\pi_B^{-r}=\frac{1}{18t}\left(\frac{5}{2}tl-\varepsilon\right)^2-\frac{\varepsilon}{6t}\left(\frac{5}{2}tl-\varepsilon\right)\right)$	$a^*=\frac{l}{2}$, $b^*=l$

Lemma 8 *Royalties licensing is better than fixed fee licensing when innovation is non drastic.*

PROOF. *If* $0<\varepsilon<\frac{8}{5}tl-\frac{2}{5}t\left(a+b\right)$ *then* $\Pi_A^r-\Pi_A^F=-\frac{\varepsilon}{6t}\left(\varepsilon-3tl\right)>0$

If $\frac{8}{5}tl-\frac{2}{5}t(a+b)<\varepsilon<2tl+t(a+b)$ *then* $\Pi_A^r-\Pi_A^F=-\frac{1}{18t}\left(\varepsilon^2-\frac{17}{2}tl\varepsilon-\frac{13}{4}t^2l^2\right)>0$ *since* $\varepsilon'=\frac{17-\sqrt{341}}{4}tl<0<2tl+t(a+b)<\varepsilon''=\frac{17+\sqrt{341}}{4}tl$.*If* $2tl+t(a+b)<\varepsilon<4tl-t(a+b)$ *then* $\Pi_A^r-\Pi_A^F=-\frac{1}{18t}\left(\varepsilon^2-\frac{15}{2}tl\varepsilon-6t^2l^2\right)>0$ *since* $\varepsilon'=\frac{15-\sqrt{321}}{4}tl<0<4tl-t(a+b)<\varepsilon''=\frac{15+\sqrt{321}}{4}tl$

Proposition 9 *Patent holding firm license its innovation under royalties regime only when innovation is intermediate. For a small innovation, it will benefit alone from the new technology and become a monopoly for a drastic innovation..*

7 Conclusion

We studied in this paper a Hotelling model on a linear city where firms are located in first and the second halfs of the city. We supposed that one of them has a patented cost reducing innovation. We studied two different licensing contracts: a per unit royalty and a fixed fee and we found that optimal locations of the two firms depend on the licensing regime and the size of the innovation. We found that on a hotelling model, no licensing is always better than fixed fee regime independently of innovation size (drastic and non drastic innovations). We showed that for a non drastic innovation, royalty licensing is better than fixed fee licensing for the patent holding firm. Results showed also that royalty licensing is better than no licensing for non drastic innovation and is optimal only when innovation is intermediate. In fact, for a small innovation, a Nash equilibrium do not exist and we can not discuss licensing regimes with only one firm on the city. Finally, when innovation is drastic, innovative firm will benefit alone of its innovation and become a monopoly.

Appendix

Innovative firm profit consists of three different functions : affine, parabolic and null functions

$$\begin{cases} \pi_A = (p_1 - c_1)\, l & \textit{If} \quad p_1 < p_2 - t(b-a) \\ \Pi_A^r = (p_1 - c_1)\, \tilde{x} & \textit{If} \quad p_2 - t(b-a) < p_1 < p_2 + t(b-a) \\ \pi_A = 0 & \textit{If} \quad p_1 > p_2 + t(b-a) \end{cases}$$

A Nash equilibrium including the two competing firms exists only when total revenue of the innovative firm on the interval $p_1 \in [p_2 - t(b-a), p_2 + t(b-a)]$ is higher than its maximal profit on the interval $p_1 \in [c_1, p_2 - t(b-a)]$. Unless, we can not discuss any licensing regime since we will not find necessarily two firms on the linear city. We distinguish between two innovation sizes:

- When $0 < \varepsilon < \frac{8}{5}tl - \frac{2}{5}t\,(a+b)$, we have $a^* = b^* = \frac{l}{2}$, and innovative firm optimal profits are :

$$\begin{cases} \pi_A^{\max} = \left(tl + \frac{2}{3}\varepsilon\right) l & \textit{If} \quad p_1 < p_2 - t(b-a) \\ \Pi_A^r = \frac{1}{18t}\,(3tl+\varepsilon)^2 + \frac{\varepsilon}{6t}\,(3tl-\varepsilon) & \textit{If} \quad p_2 - t(b-a) < p_1 < p_2 + t(b-a) \\ \pi_A = 0 & \textit{If} \quad p_1 > p_2 + t(b-a) \end{cases}$$

$$\Pi_A^r - \pi_A^{\max} = -\frac{1}{9t}\left(\varepsilon^2 - \frac{3}{2}tl\varepsilon + \frac{9}{2}t^2l^2\right) < 0 \text{ (since } \Delta = -\frac{63}{4}t^2l^2 < 0)$$

We conclude that when innovation is small, a Nash equilibrium do not exist for a royalty licensing and firm A has not interest to be on the price interval $p_1 > p_2 - t(b-a)$ since being on the interval $p_1 < p_2 - t(b-a)$ is better and it can makes higher profits than the other interval. Since in the interval $p_1 < p_2 - t(b-a)$ firm B leave the market so we have no license here.

- However, when $\frac{8}{5}tl - \frac{2}{5}t(a+b) < \varepsilon < 4tl - t(a+b)$, we have $a^* = \frac{l}{2}$ and $b^* = l$, and innovative firm optimal profits are :

$$\begin{cases} \pi_A^{\max} = \left(\frac{1}{3}tl + \frac{2}{3}\varepsilon\right) l & \textit{If} \quad p_1 < p_2 - t(b-a) \\ \Pi_A^r = \frac{1}{18t}\left(\frac{7}{2}tl + \varepsilon\right)^2 + \frac{\varepsilon}{6t}\left(\frac{5}{2}tl - \varepsilon\right) & \textit{If} \quad p_2 - t(b-a) < p_1 < p_2 + t(b-a) \\ \pi_A = 0 & \textit{If} \quad p_1 > p_2 + t(b-a) \end{cases}$$

$\Pi_A^r - \pi_A^{\max} = -\frac{1}{9t}\left(\varepsilon^2 - \frac{5}{4}tl\varepsilon + \frac{25}{8}t^2l^2\right) > 0$ (since $\varepsilon' = -\frac{5}{4}tl < 0 < \varepsilon'' = \frac{5}{2}tl = 4tl - t(a+b)$)

Nash equilibrium exists here under royalty licensing for an intermediate innovation.

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